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CASE
Conference in Advancements in Sustainable Engineering

A stochastic Implementation of Emission Tomography

Theodoros Leontiou, Yiannis Parpottas, Elena Ttofi, Anna Frixou, Efstathios Stiliaris, Charalambos Chrysostomou and Costas N Papanicolas

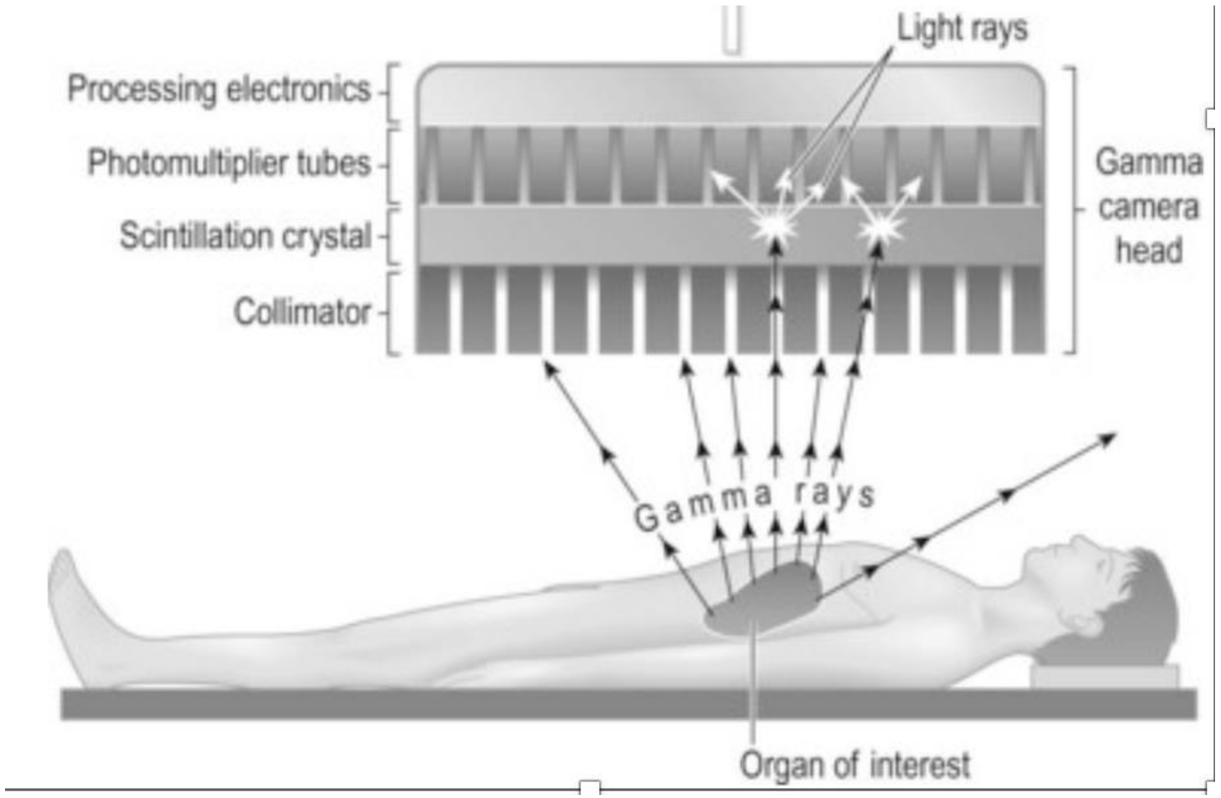
- Introduction to Emission Tomography
- The Inverse Problem – Image Reconstruction
- Stochastic Implementation
- Application to Thyroid Imaging

Gamma rays

Emission tomography, is a powerful imaging technique used to visualize physiological functions of body organs.

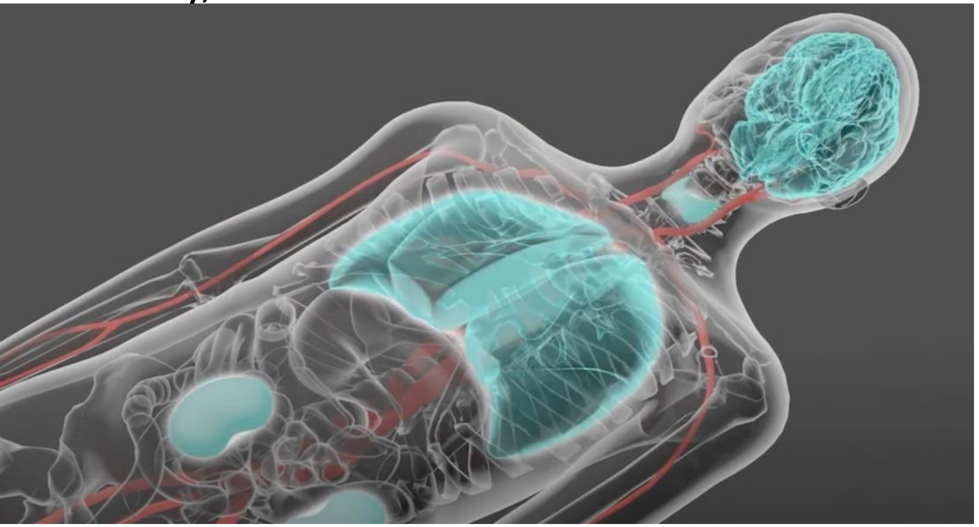
It relies on the detection of gamma rays emitted by radiotracers administered to the patient.

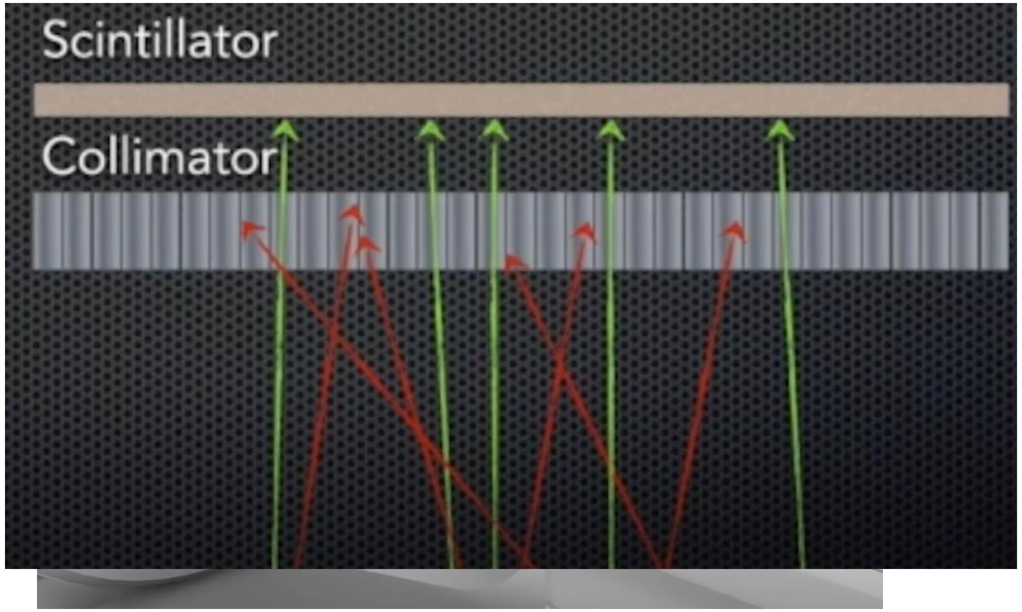
Why gamma rays? They can penetrate through soft tissue, muscles and bone and reach the detection camera.



SPECT – Single Photon Emission Computerized Tomography

The patient is injected with a radioactive biomolecule that binds at a certain region of the body, such a tumour.





The radioactive biomolecule emits high energy photons which travel outside the patients body towards a gamma camera.

A lead collimator design is used to ensure that that photons are only detected if the travel perpendicular to the gamma camera.

SPECT, PET and CT

SPECT (Single Photon Emission Computed Tomography)

– Uses gamma-emitting isotopes

– Provides 3D images via rotating gamma camera

PET (Positron Emission Tomography)

– Uses positron-emitting isotopes

– Detects annihilation photons (511 keV) in pairs

PET/CT and SPECT/CT

– Combines functional imaging with anatomical CT for better localization

Applications: Cardiology

Myocardial perfusion imaging or scanning illustrates the function of the heart muscle

- Radiopharmaceutical:

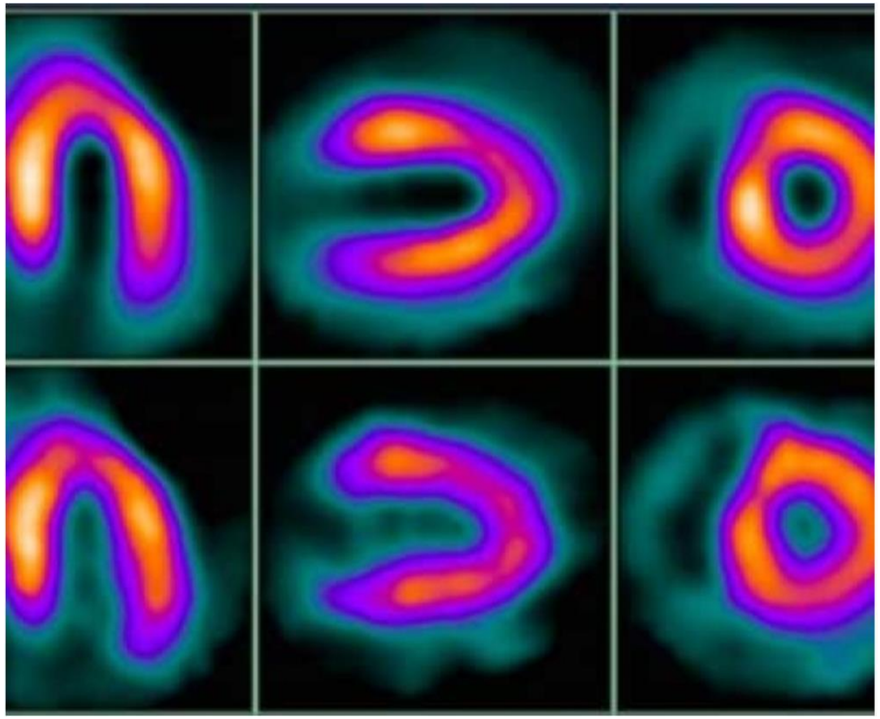
Tc-99m

- Typical Dosage:

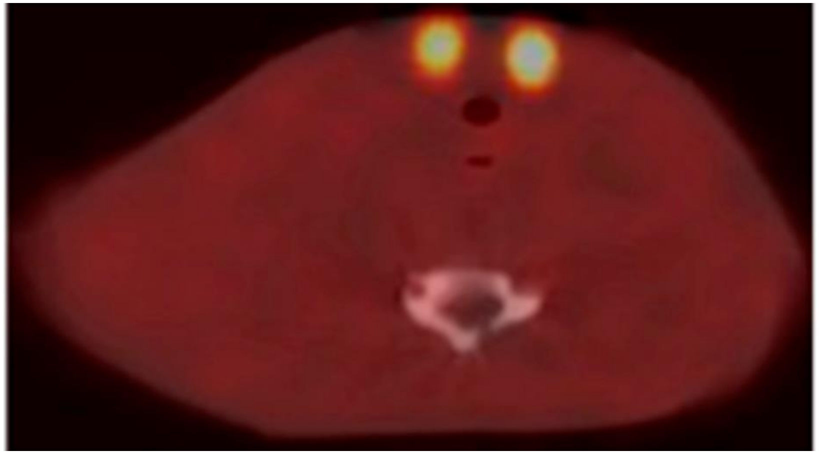
370–1110 MBq (10–30 mCi)

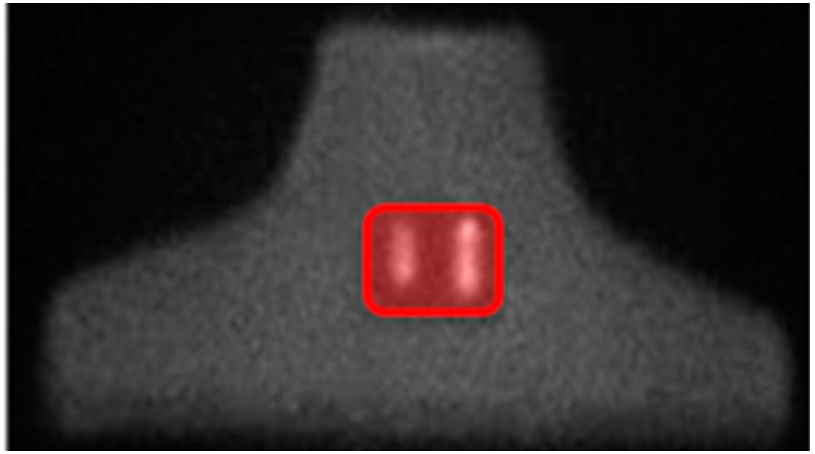
- Gamma Photon Energy:

140 keV



Applications: Endocrinology

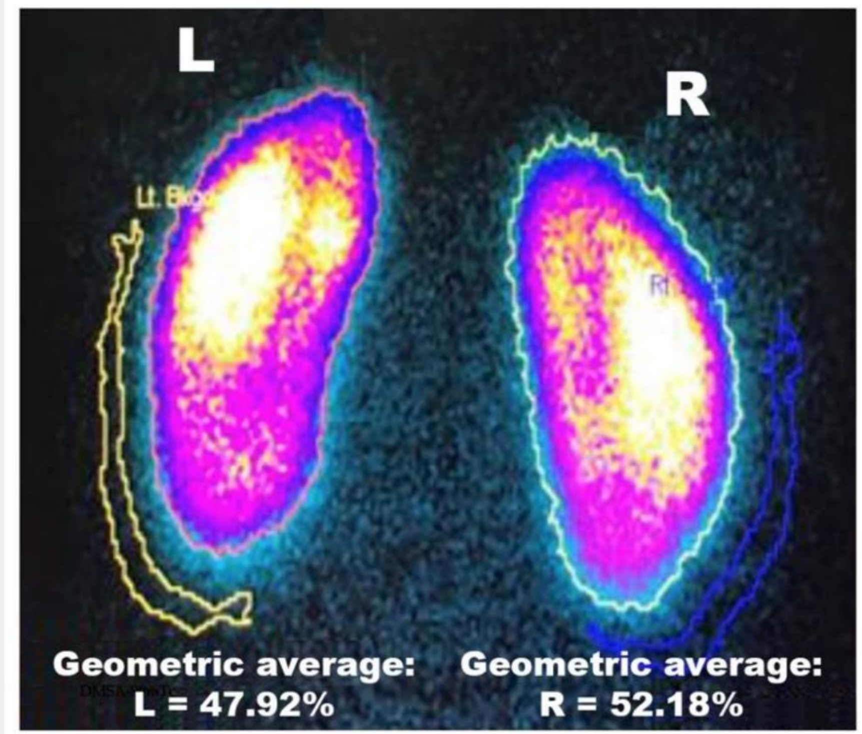




Diagnostic thyroid Imaging (I123) and thyroid cancer therapy (I131)

Applications: Nephrology

- Radiopharmaceutical containing Tc-99m
- Typical Dosage: 370 MBq (10 mCi)
- Gamma Photon Energy: 140 keV



SPECT investigation of Kidney function

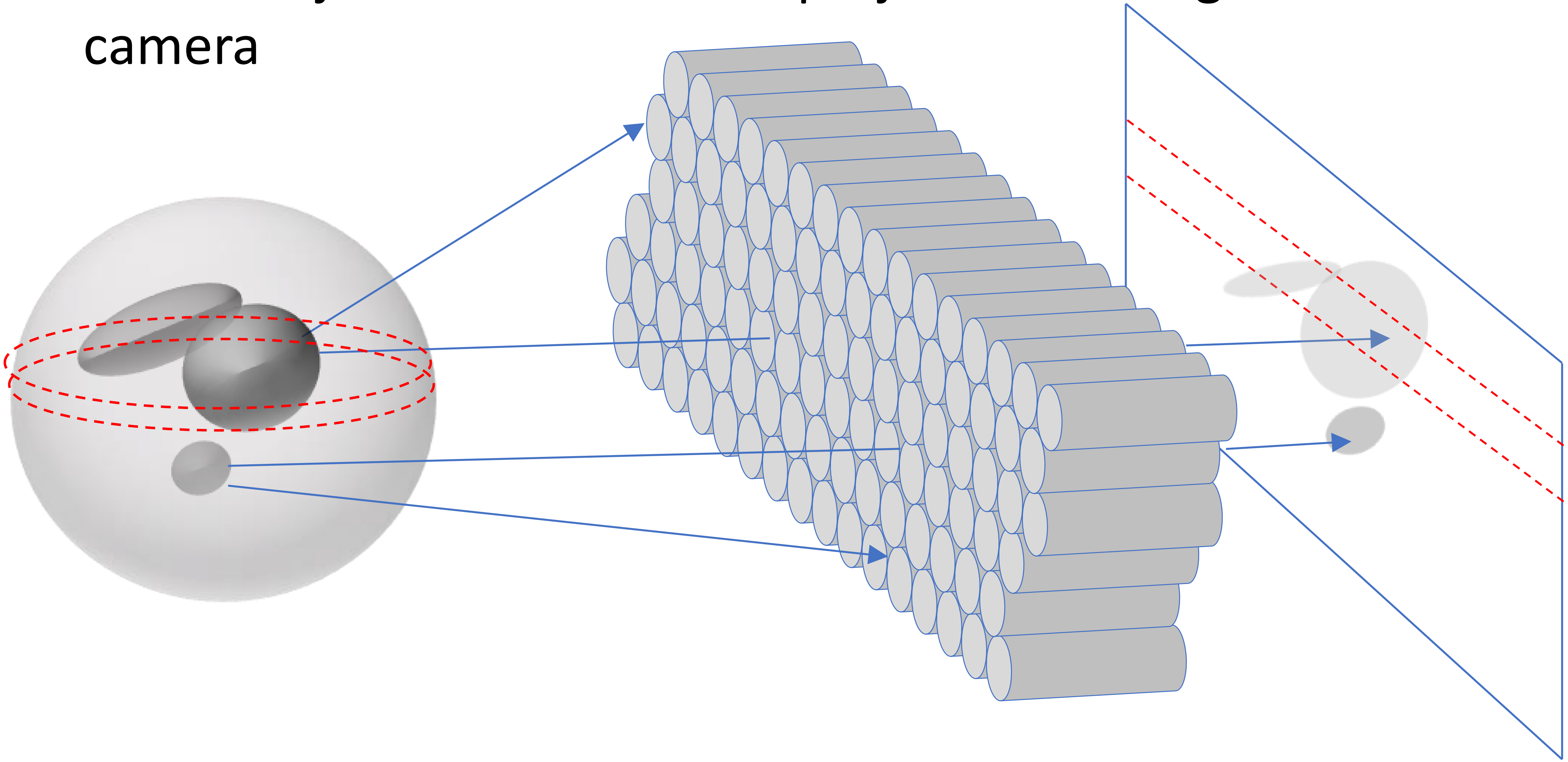
Applications: Bone Imaging

A nuclear medicine whole-body bone scan is used in evaluations of various bone-related pathologies, such as bone infections, or the spread of cancer to the bone.

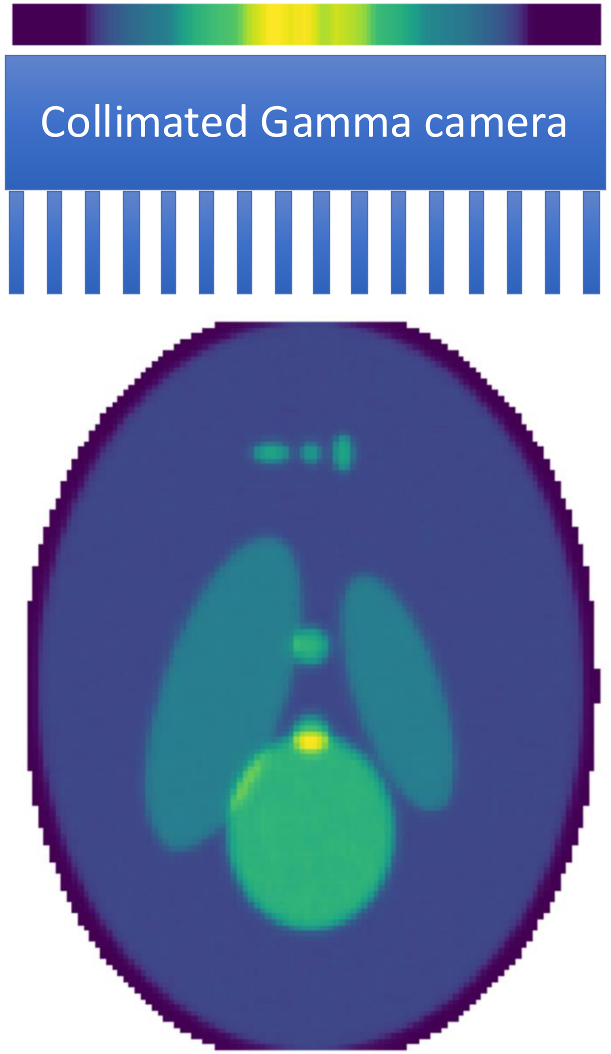
- Radiopharmaceutical: Tc-99m MDP
- Typical Dosage: 740 MBq (20 mCi)
- Gamma Photon Energy: 140 keV

Projections

- For 3D object 2D sections are projected to the gamma camera

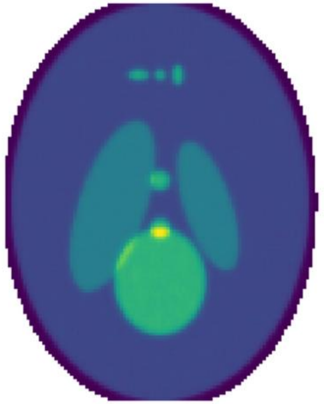


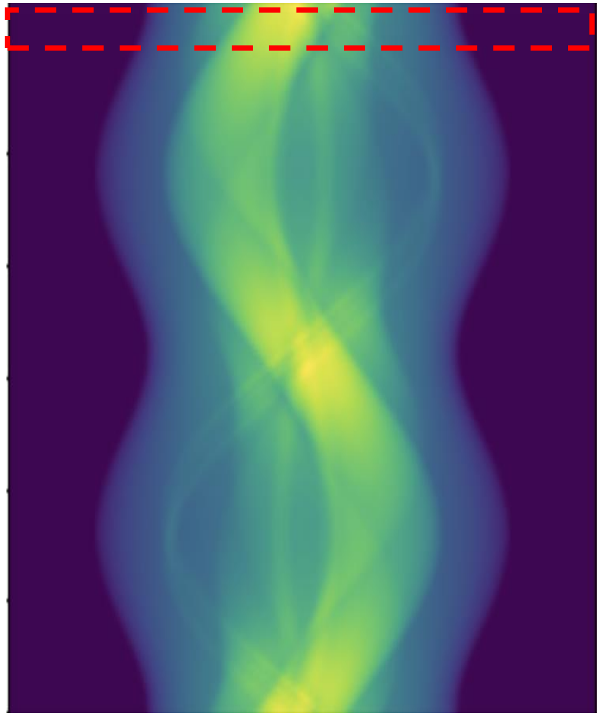
Projections



- For each section through the actual 3D radiation source the gamma camera measures the total number of photons reaching the detector (counts)
- Information about the actual size is lost for a particular orientation of the camera

Projections -> Sinograms





Generated sinogram

- By using different projections, usually by circulating the target a ‘sinogram’ is generated for each section of the target
- Reconstruction algorithms are then used to recover the original radiation source

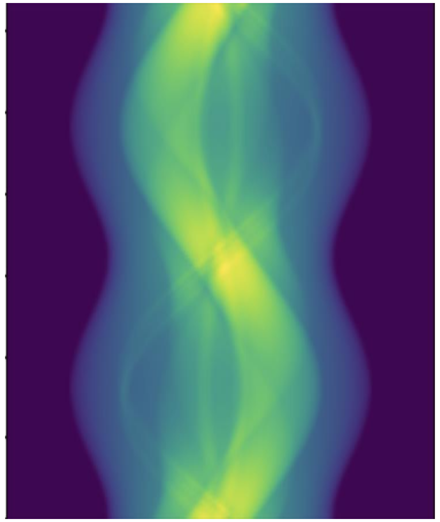
Image Reconstruction

Radiotracer injection – Uptake & Decay (γ-ray emission)

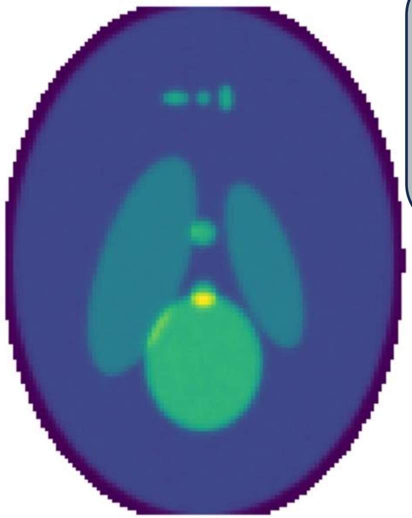
Photon Propagation through Body

Detection by Collimated Gamma Camera

Projection Data (Sinograms)



Inverse Problem: **Image Reconstruction**



Introduction to Emission Tomography

The Inverse Problem – Image Reconstruction

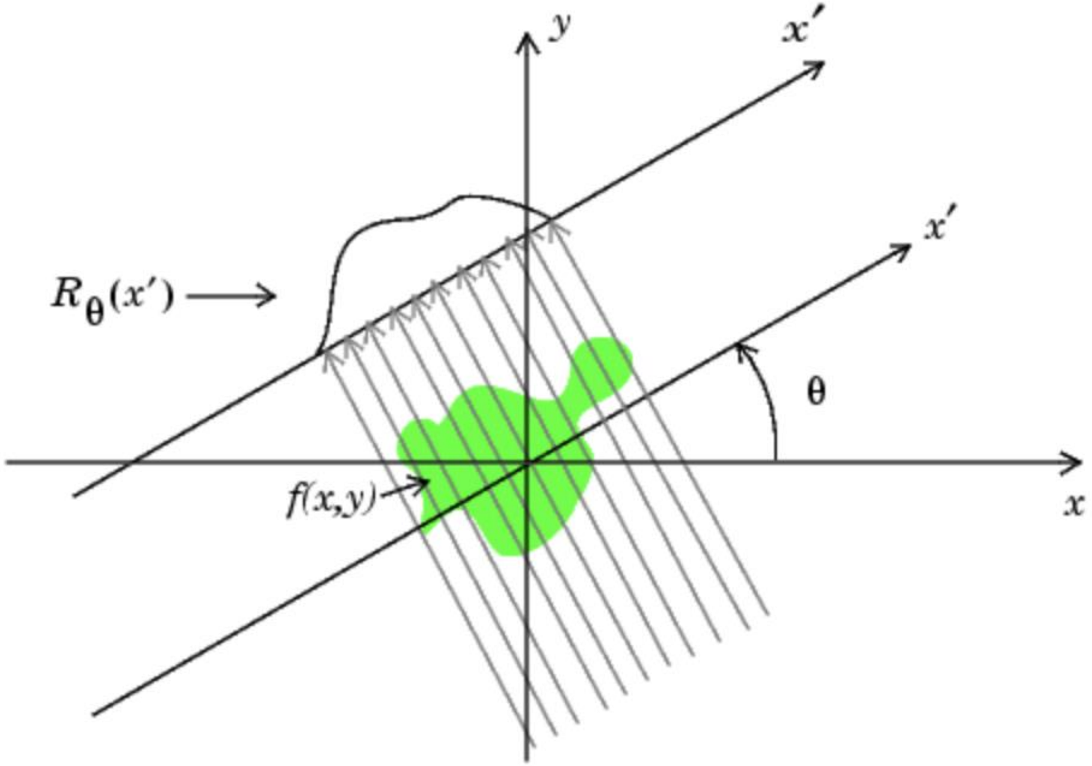
Stochastic Implementation

Application to Thyroid Imaging

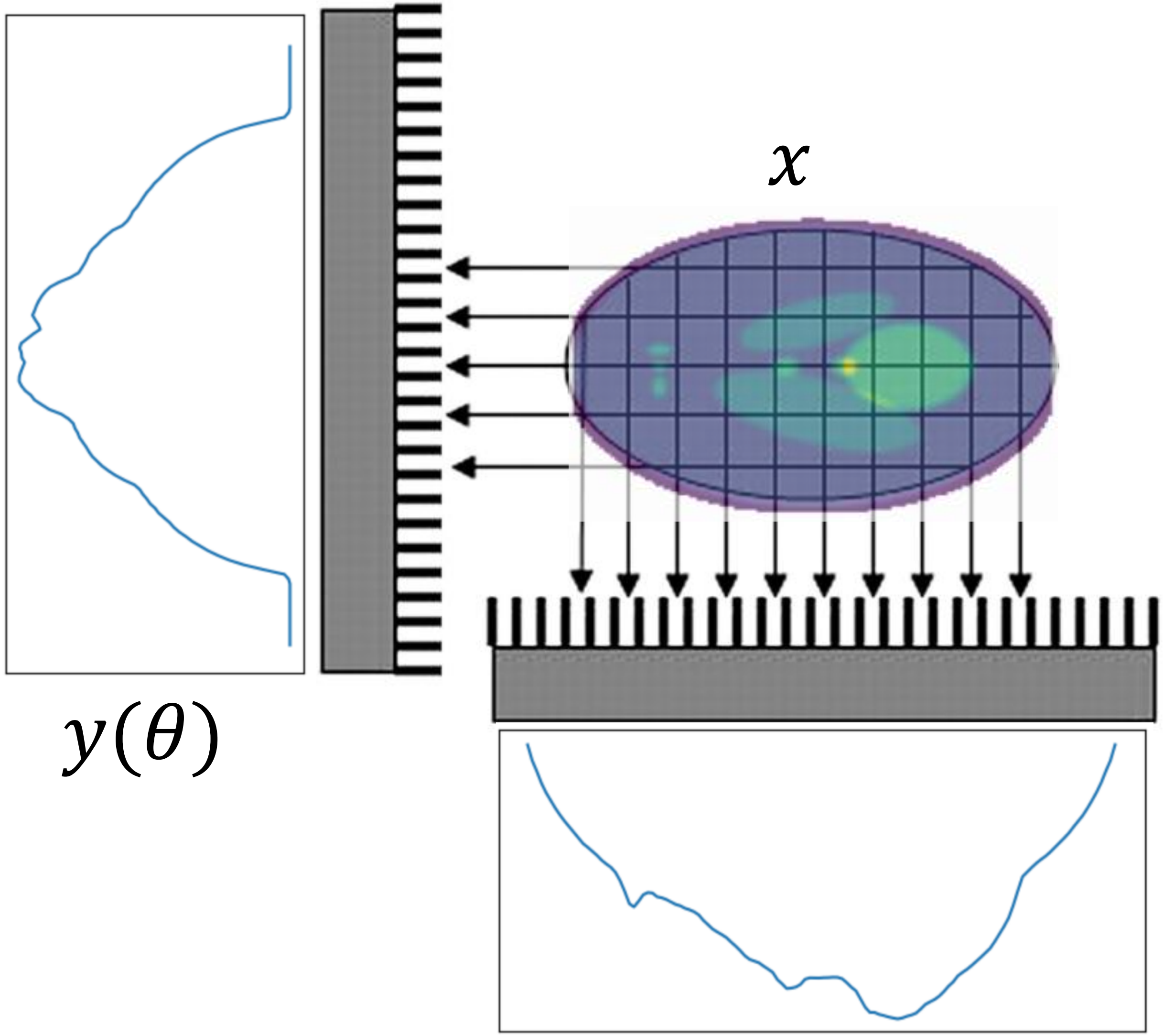
Projection Matrix

The inverse problem completely relies on having a **well defined model** which represents the projections as a function of the source

The source is unknow and needs to be determined by inverting this function

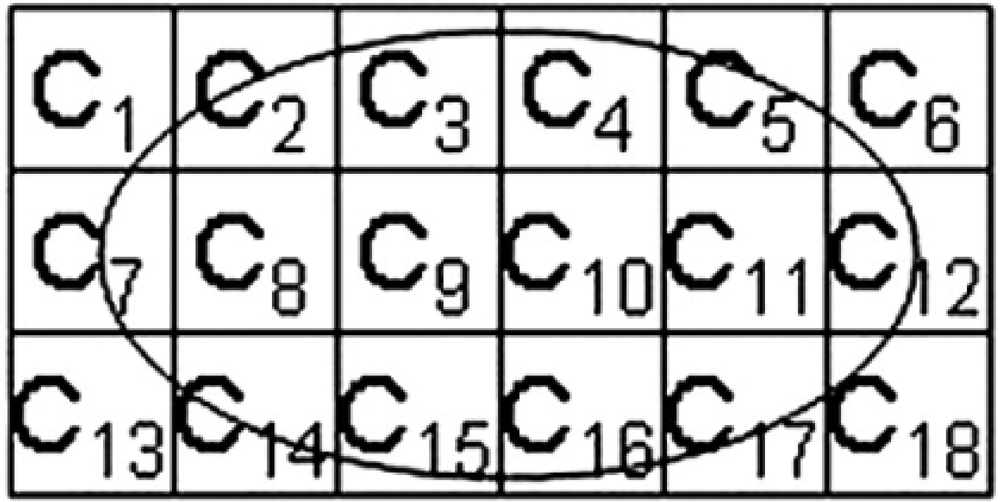


Projection Matrix



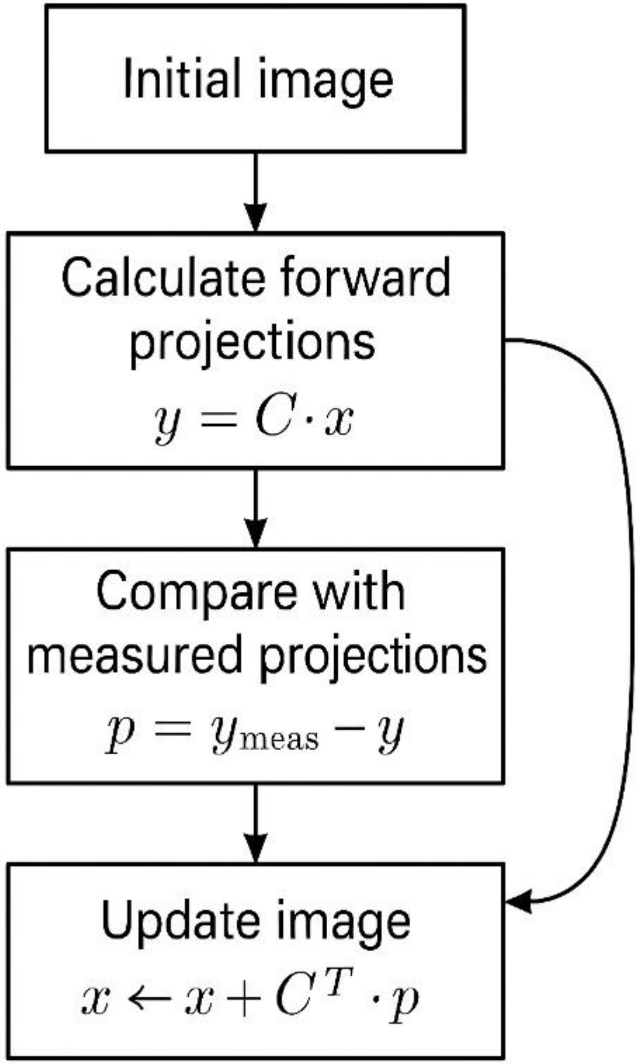
$y(\theta)$

Projection Matrix



$$y(\theta)=C(\theta).x$$

Iterative Reconstruction Methods



- **Initial Image**: Start with a basic estimate of the image
- **Forward Projections**: Simulate projections using the current image estimate.

$$y = C \cdot x$$

- **Compare** with Measured Projections:

  Compute the difference between measured and simulated projections.

Update Image:

Refine the image using the backprojected residual.

$$x \leftarrow x + C^T \cdot p$$

- **Iterate** until the residual is small or a stopping condition is met.

Stochastic Implementation

Stochastic Gradient Descent in SPECT Reconstruction

- Loss function (χ^2 discrepancy):

$$L = \Sigma (y_i - (Px)_i)^2 / (y_i + \varepsilon)$$

- **Adam Optimizer (Adaptive Moment Estimation)**

Adam is one of the most widely used gradient-based optimization algorithms in machine learning. It improves upon standard stochastic gradient descent (SGD) by combining the benefits of two earlier methods:

Momentum (which accumulates a moving average of past gradients, smoothing updates).

RMSProp (which rescales updates by a moving average of past squared gradients, adapting the learning rate per parameter).

- Stabilization:

– Gaussian smoothing (noise suppression)

– Additional constraints

– Resolution-aware updates (coarse → fine)

Why are stochastic methods attractive today?

- Machine learning revolution → robust optimization tools.
- GPU acceleration:
  - Parallel linear algebra & convolutions
  - Automatic differentiation
  - Scales to 10^6+ voxel reconstructions
- No longer an exotic alternative:
  - Preserves statistical fidelity
  - Clinically viable runtimes

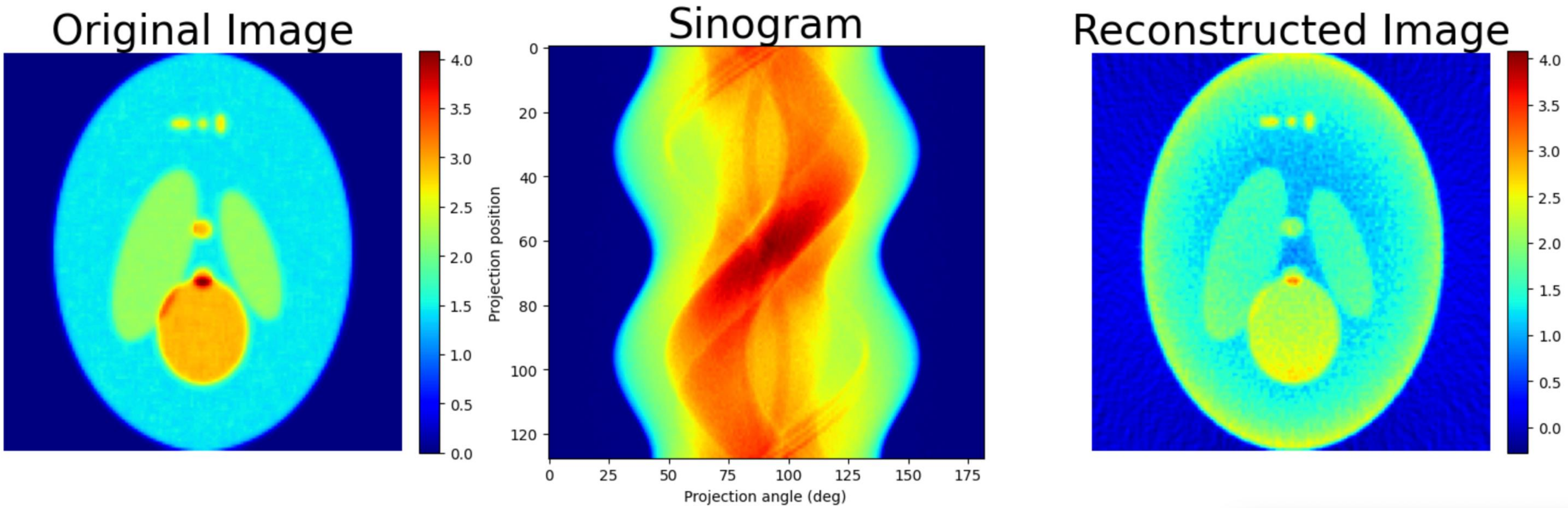
Attenuation

Photons emitted within the body are attenuated (absorbed or scattered) before reaching the detector

This results in reduced image quality and misinterpretation of activity concentrations. This leads to underestimation of activity in deeper tissues

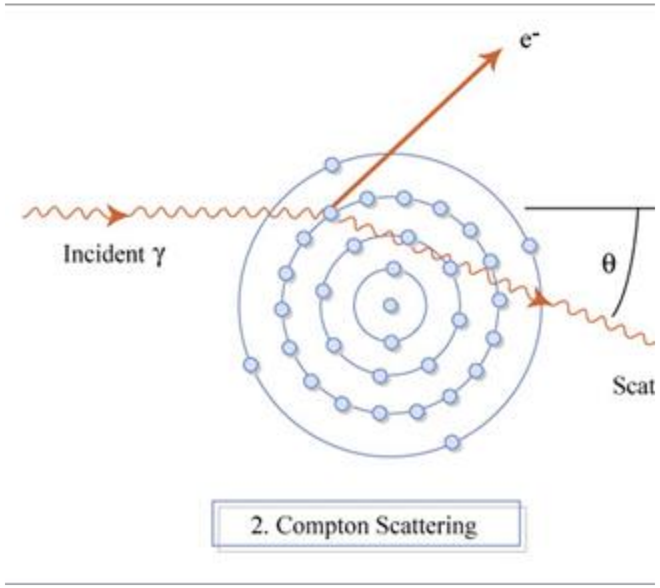
Accurate modelling of attenuation is essential for high-quality, quantitative imaging results.

Attenuation

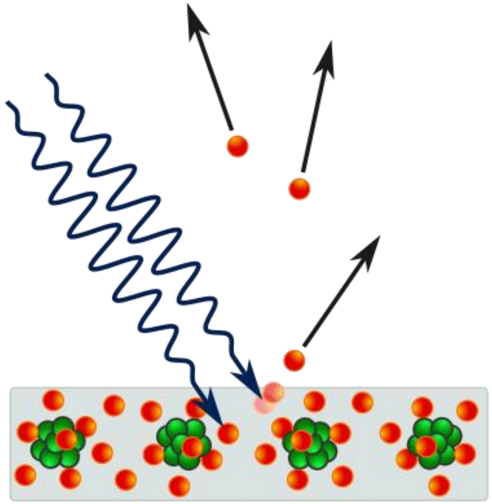


Attenuation leads to underestimation of activity in deeper tissue

Attenuation



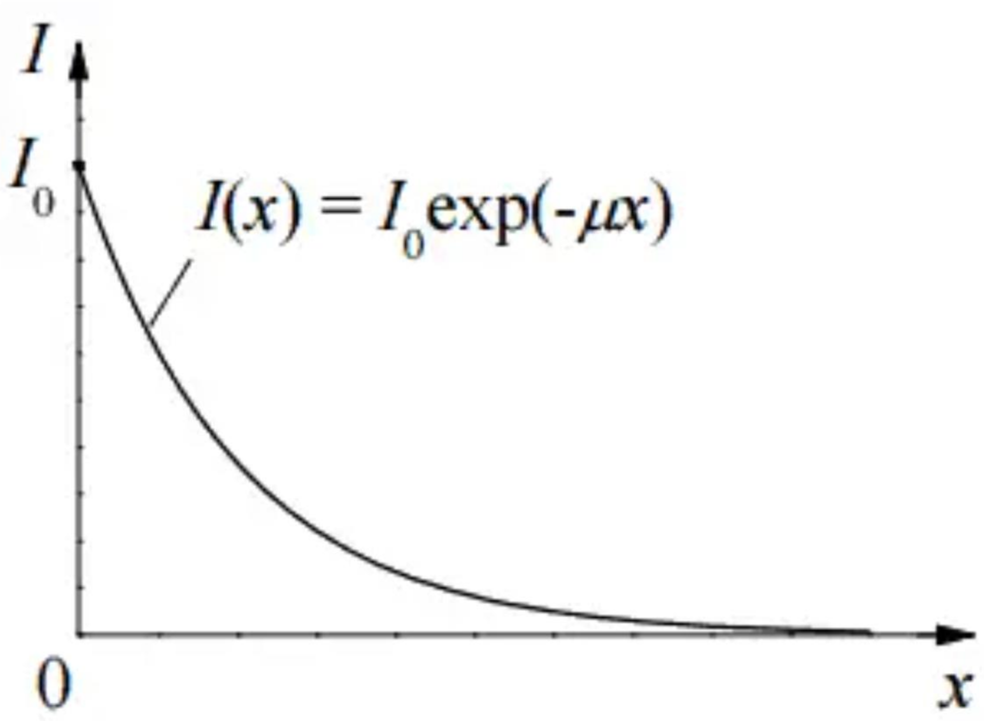
Compton Scattering



Photoelectric Effect

Attenuation

Beer–Lambert Law



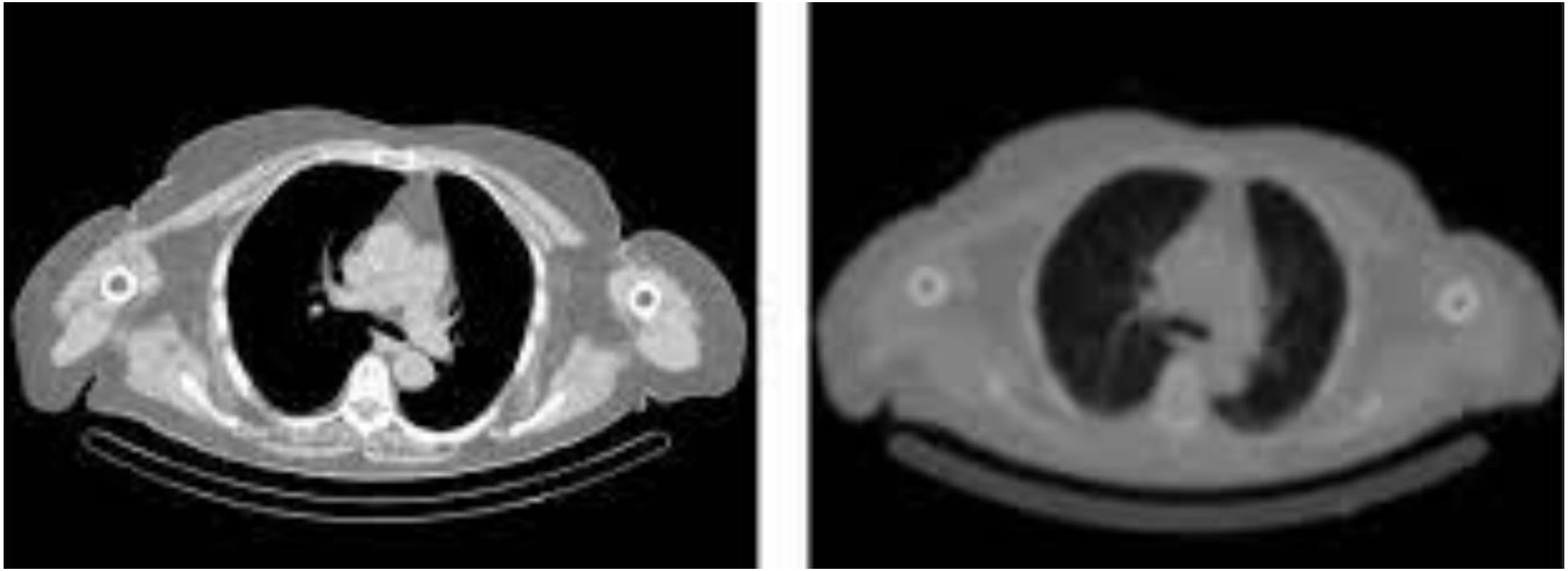
Dependence of gamma radiation intensity on absorber thickness

- The occurrence of any attenuation event is characterized by a fixed probability
- The sum of these probabilities is called the **linear attenuation coefficient**:

$$\mu = \tau_{(photoelectric)} + \sigma_{(Compton)}$$

Absorber	100 keV	200 keV	500 keV
Air	0.0195/m	0.0159/m	0.0112/m
Water	0.167/cm	0.136/cm	0.097/cm
Carbon	0.335/cm	0.274/cm	0.196/cm

Attenuation Map



- The attenuation coefficient for the human body is non-uniform $\mu=\mu(x,y)$
- High resolution CT scans can be used in to obtain the **attenuation map**

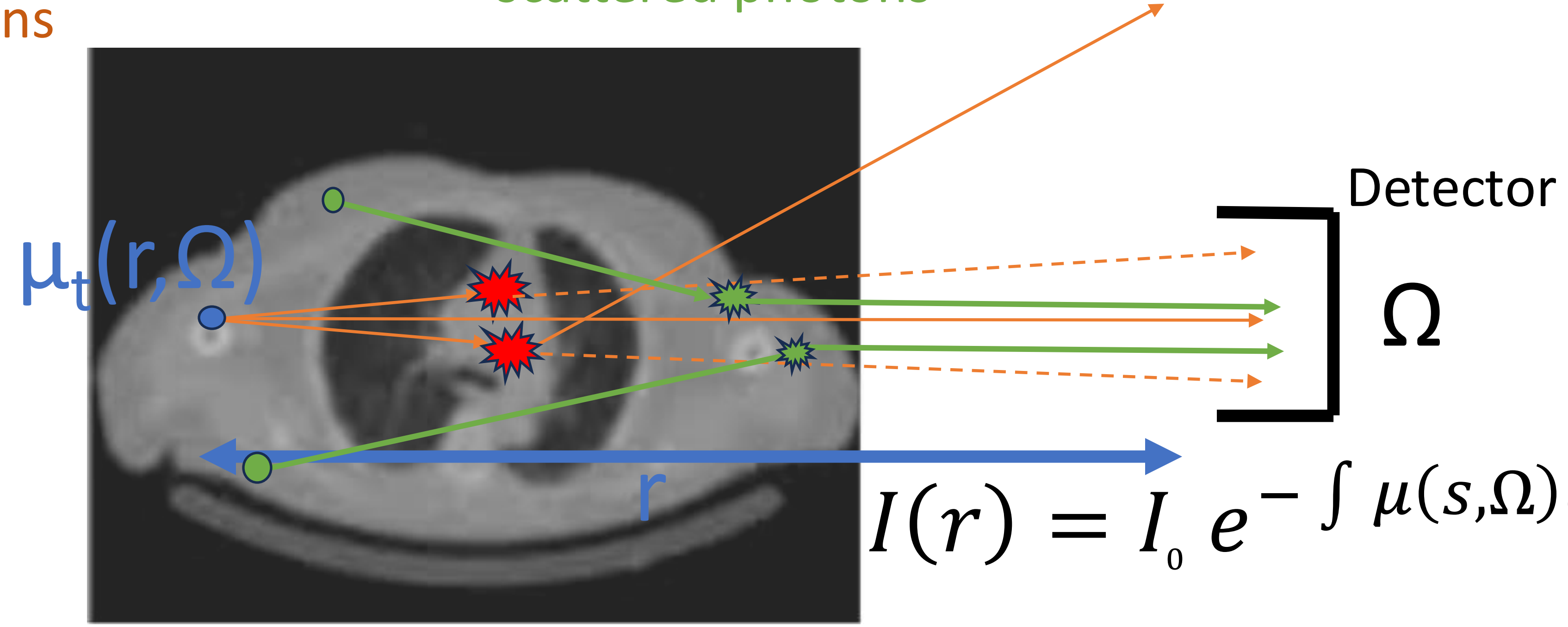
Attenuation Model

For a more complete physical model — especially in **inhomogeneous or scattering media** — we use the **Boltzmann transport equation** for photons:

$$\frac{dI(\mathbf{r},\hat{\Omega})}{ds} = -\mu_t(\mathbf{r})I(\mathbf{r},\hat{\Omega}) + \int_{4\pi} \mu_s(\mathbf{r},\hat{\Omega}',\hat{\Omega})I(\mathbf{r},\hat{\Omega}')\,d\hat{\Omega}'$$

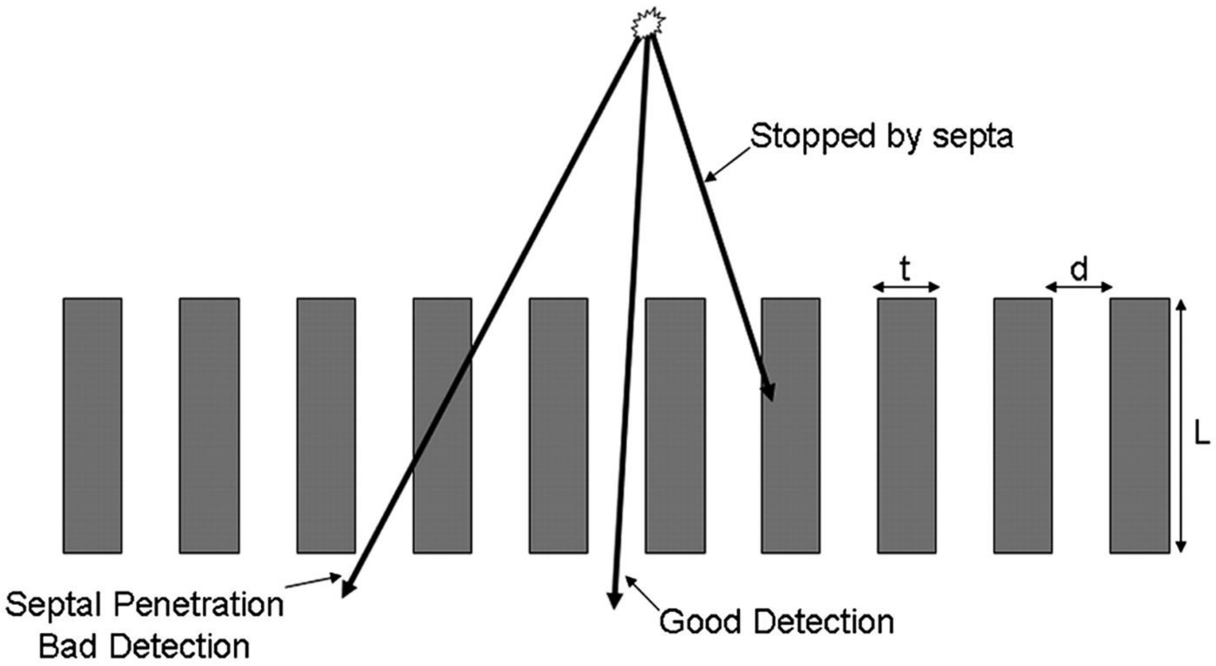
primary photons

scattered photons



$$I(r) = I_0\, e^{-\int \mu(s,\Omega)ds}$$

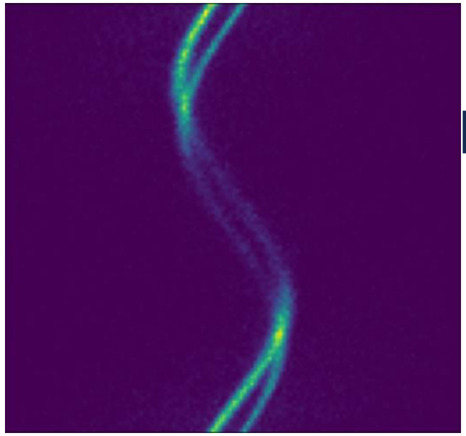
Attenuation Model



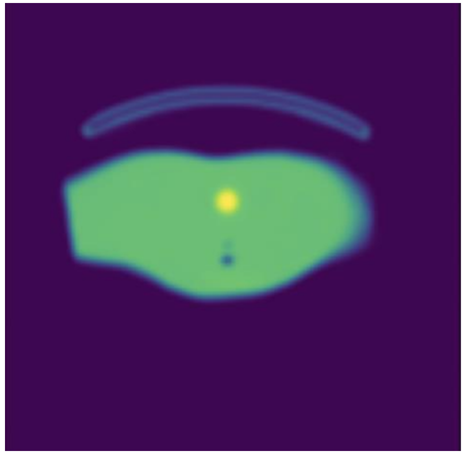
- The collimator is a slab of dense material (usually lead) with many parallel holes.
- These holes allow gamma photons traveling in specific directions to reach the detector.
- The septa are the walls between the holes that block photons coming in at an angle.
- Septal penetration leads to bad detector signal

Ideal Reconstruction Method

Measured Sinogram



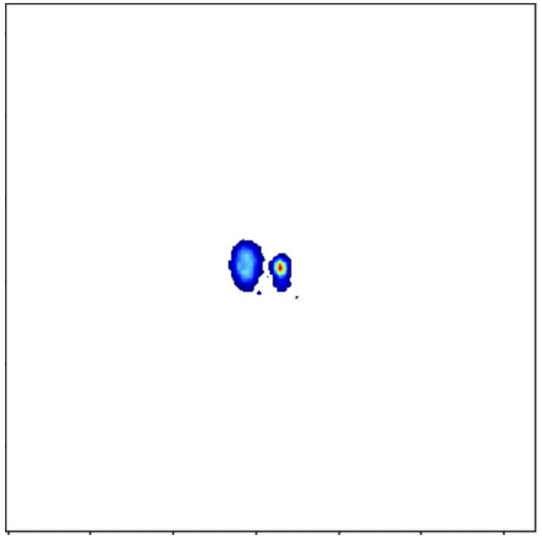
Attenuation Mad from CT



Inverse Problem: **Image Reconstruction**

Projection Matrix with Attenuation

Reconstructed Image



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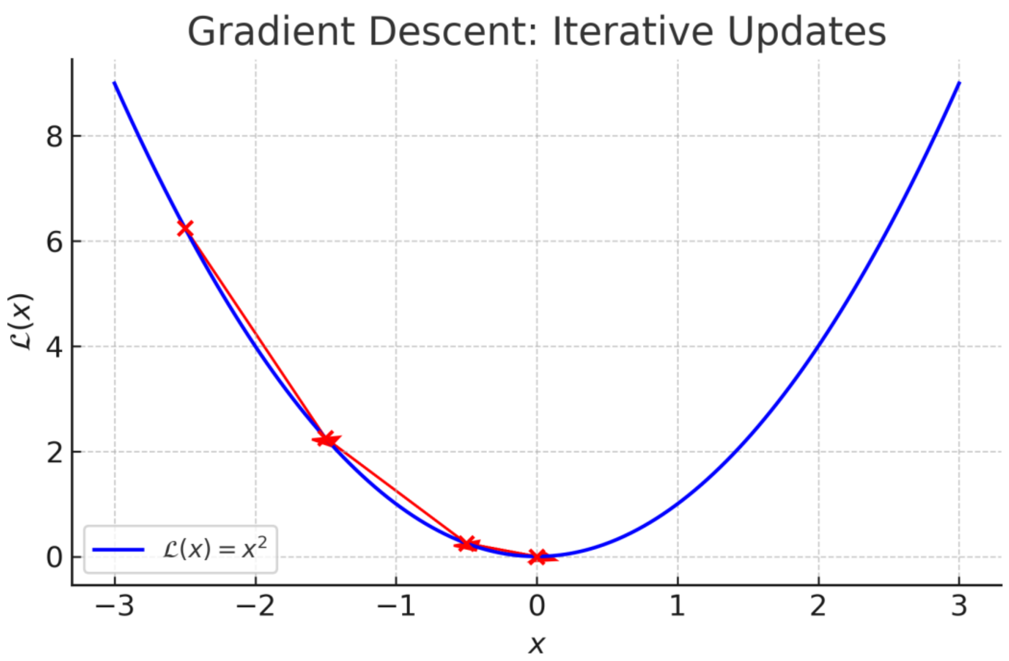
How Gradient Descent Works

- Goal: minimize a loss function L(x).
- Start from an initial guess x_0.
- Update rule:
- $x_{t+1} = x_t - \eta \nabla L(x_t)$

where η is the learning rate.



- Intuition: take steps “downhill” in the loss landscape.
- Works well in high dimensions because updates are local and efficient.

Gradient Descent in SPECT Reconstruction

- Loss function (χ^2 discrepancy):

$$L = \Sigma (y_i - (Px)_i)^2 / (y_i + \varepsilon)$$

- **Adam Optimizer (Adaptive Moment Estimation)**

Adam is one of the most widely used gradient-based optimization algorithms in machine learning. It improves upon standard stochastic gradient descent (SGD) by combining the benefits of two earlier methods:

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- Stabilization:
  - Gaussian smoothing (noise suppression)
  - Additional constraints
  - Resolution-aware updates (coarse → fine)

Why SGD is Attractive Today

- Machine learning revolution → robust optimization tools.
- GPU acceleration:
  - Parallel linear algebra & convolutions
  - Automatic differentiation
  - Scales to 10^6+ voxel reconstructions
- No longer an exotic alternative:
  - Preserves statistical fidelity
  - Clinically viable runtimes

Hamiltonian Monte Carlo (HMC): The Concept

- HMC reformulates sampling as simulating a physical system.
- Introduces auxiliary momentum variables p.
- Defines Hamiltonian: $H(x, p) = U(x) + K(p)$.
-    – Potential energy $U(x) = \chi^2(x)/2$ (from data likelihood)
-    – Kinetic energy $K(p) = \frac{1}{2} p^T M^{-1} p$
- Samples drawn by simulating Hamilton’s equations of motion.
- Provides efficient exploration of high-dimensional distributions.

HMC for SPECT Reconstruction

- Likelihood:
- $\Pi(y|x) \propto \exp(-\frac{1}{2} \Sigma (y_i - (Px)_i)^2 / (y_i + \varepsilon))$.

- Potential energy:
- $U(x) = \Sigma (y_i - (Px)_i)^2 / (2(y_i + \varepsilon))$.

- Gradient of U(x) guides proposals.
- Leapfrog integrator used for stable numerical simulation.
- Metropolis acceptance step ensures detailed balance.

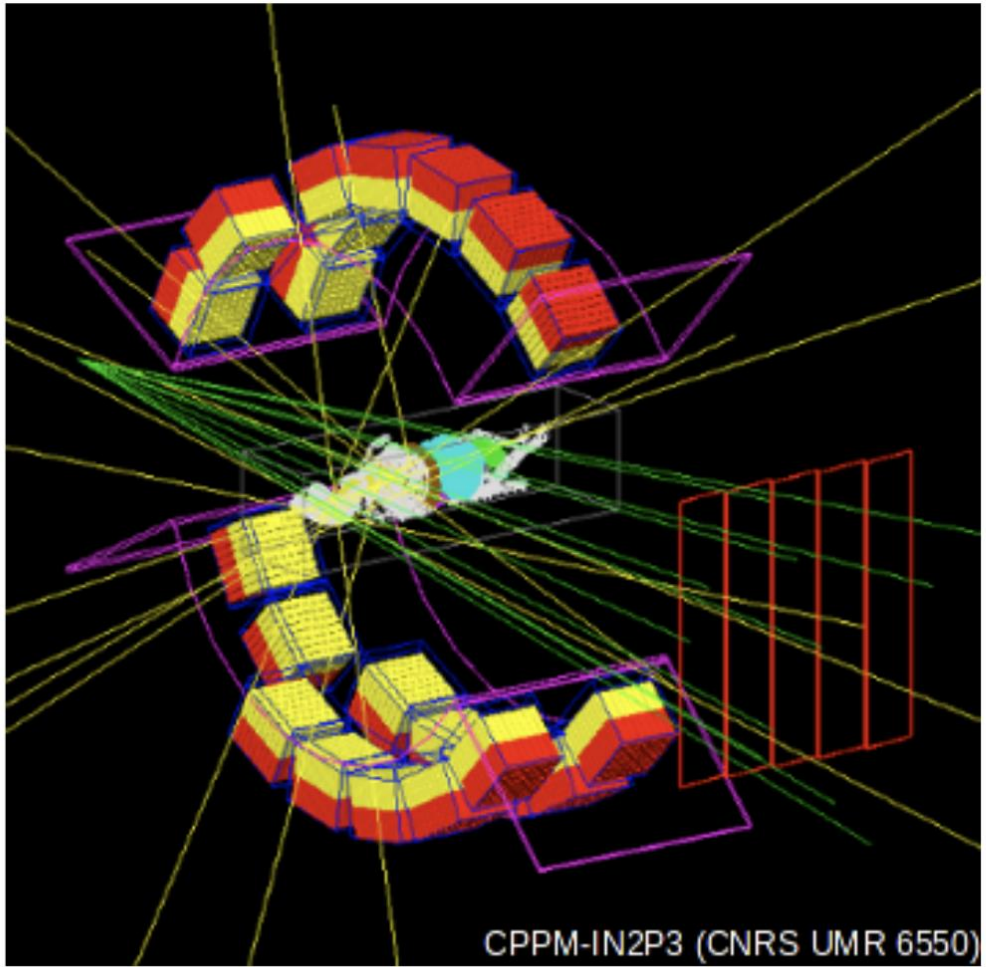
Why HMC is Attractive Today

- Overcomes random-walk limitations of basic MCMC.
- Enables long-range, informed moves in parameter space.
- Provides ensembles of reconstructions → expectation values.
- Naturally produces voxel-wise uncertainty (σ maps).

- GPU acceleration:
  – Parallel forward projections & gradient evaluations
  – Efficient simulation of Hamiltonian dynamics

- Clinically relevant: scalable to high-resolution images.

Demonstration using Simulations

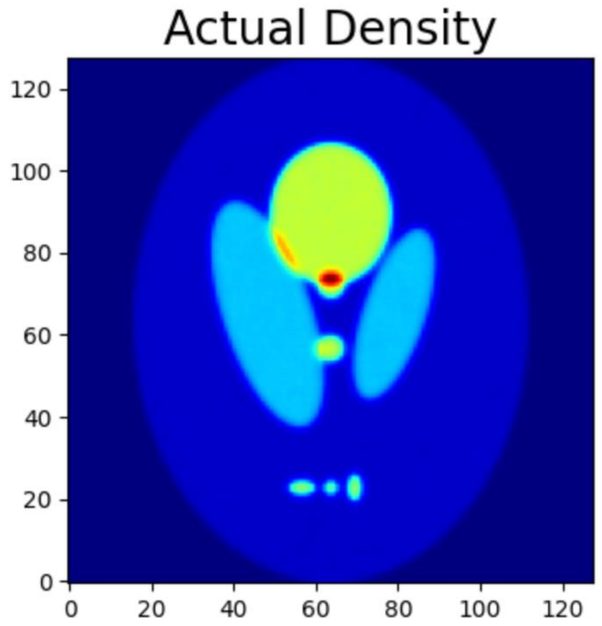


GEANT4: A **Monte Carlo simulation toolkit** developed by an international collaboration, originally at CERN.
Used for simulating the passage of particles through matter.
Covers a wide range of physics:

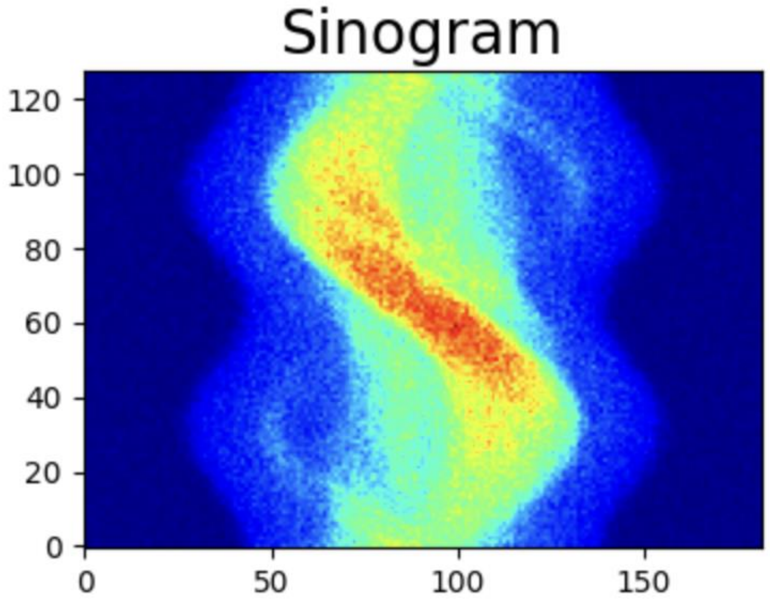
- Electromagnetic interactions
- Hadronic interactions
- Optical photons
- Radioactive decay, etc.

Geant4 Application for Tomography Emission (GATE) is an advanced open-source software developed by the international OpenGATE collaboration and dedicated to numerical simulations in medical imaging and radiotherapy.

Demonstration using Simulations

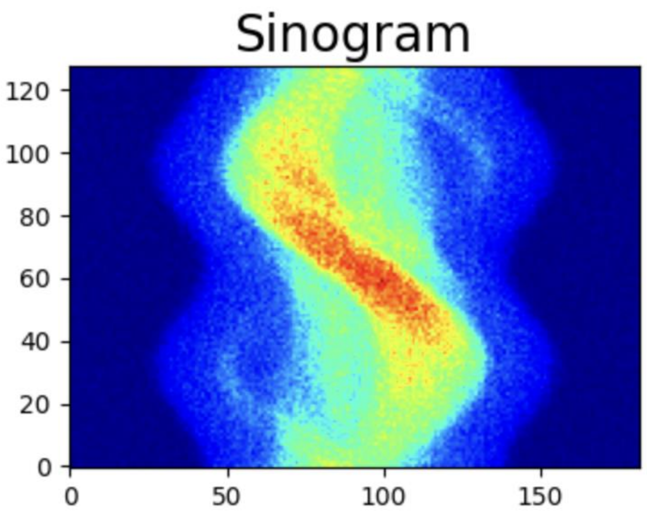


GATE simulation using detector with collimator

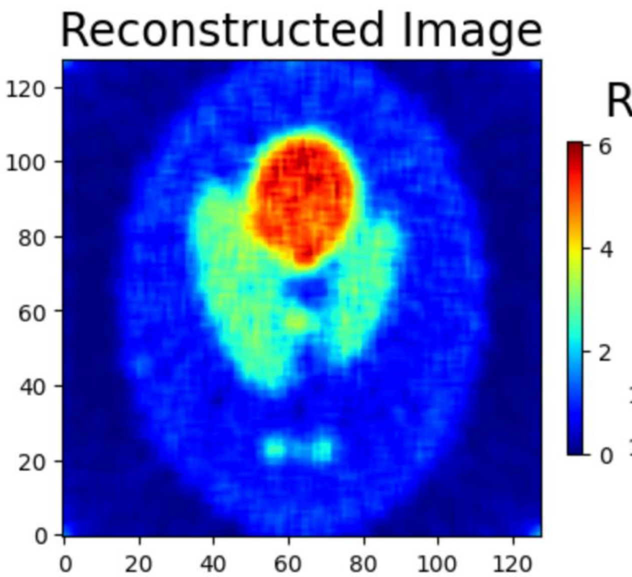


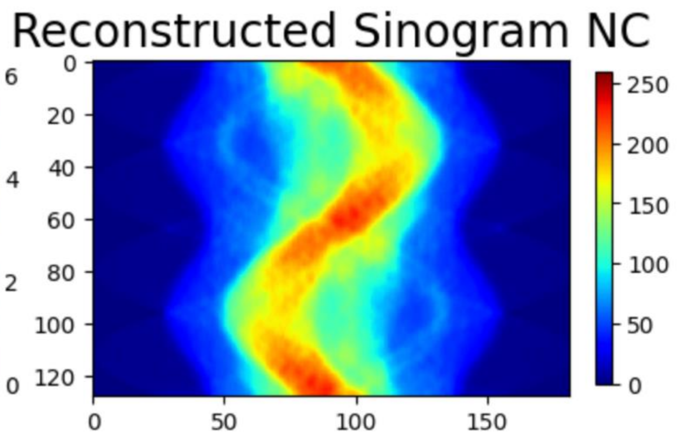
Detector signal using 128 projections

Demonstration using Simulations

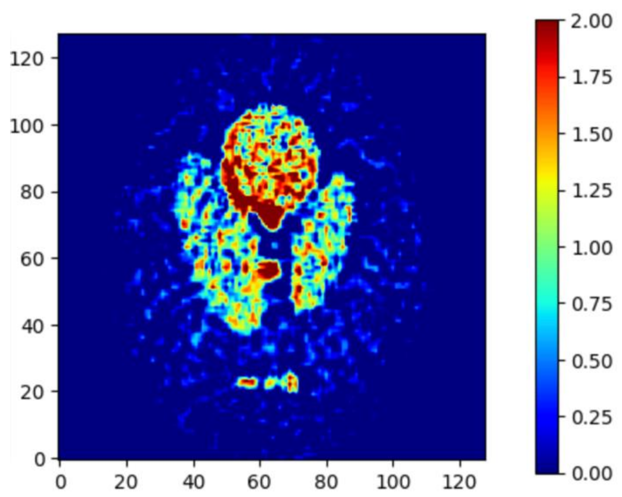


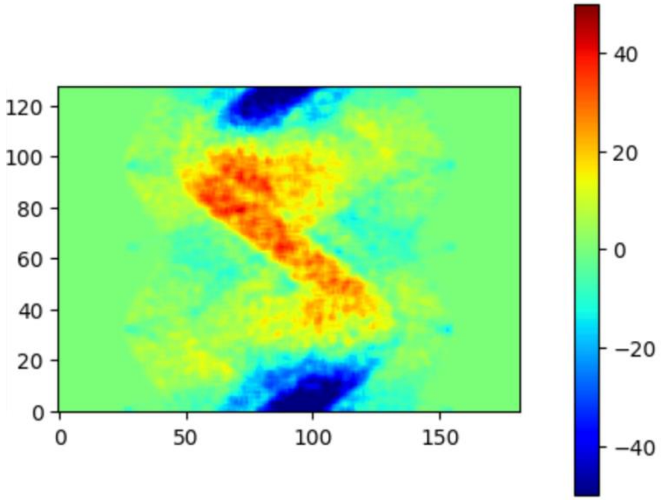
Reconstruction without Attenuation correction



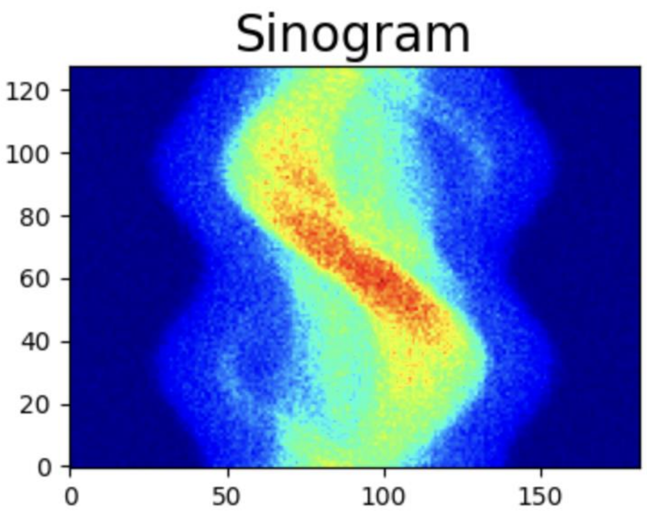


Differences between actual and reconstructed

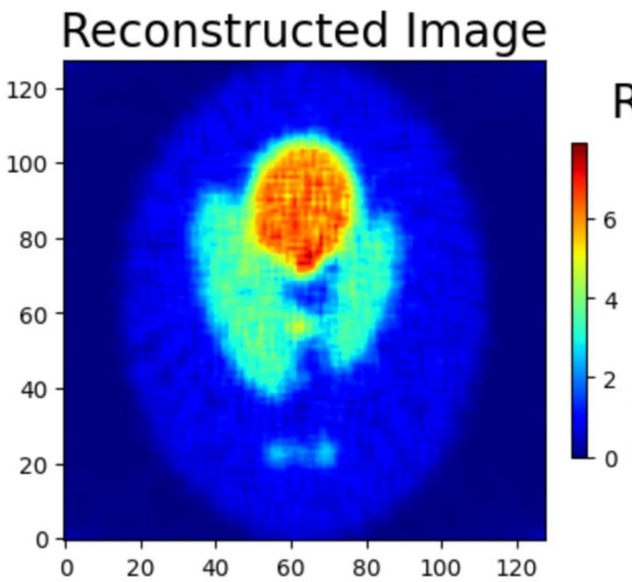


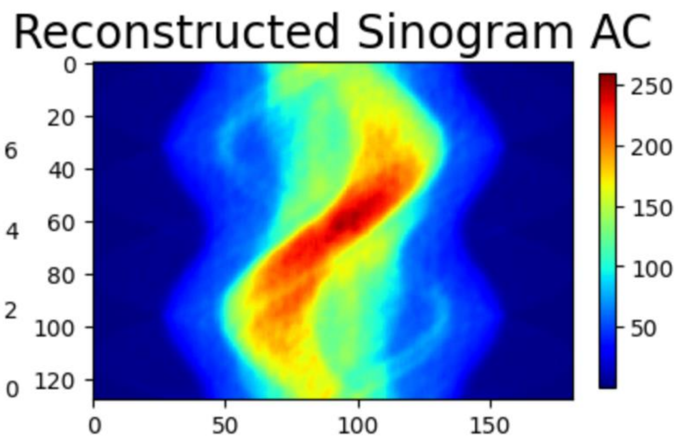


Demonstration using Simulations

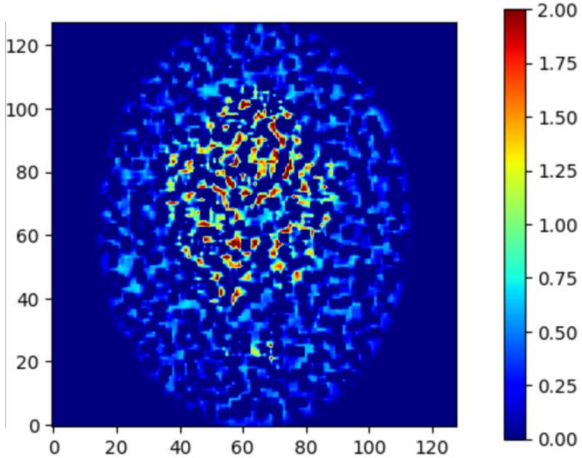


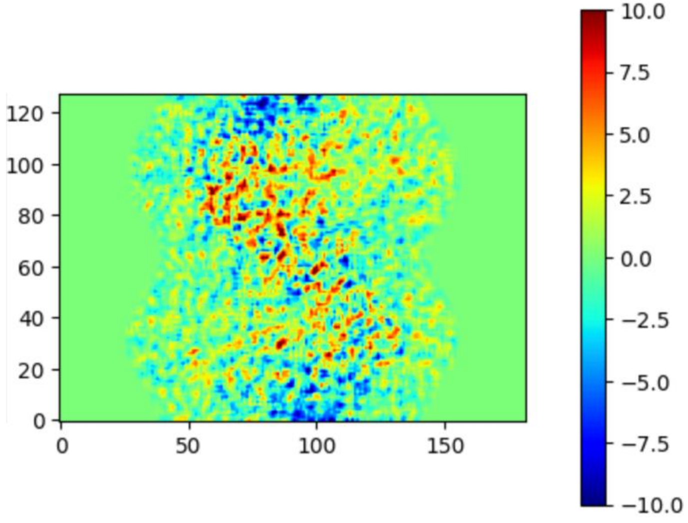
Reconstruction with Attenuation correction



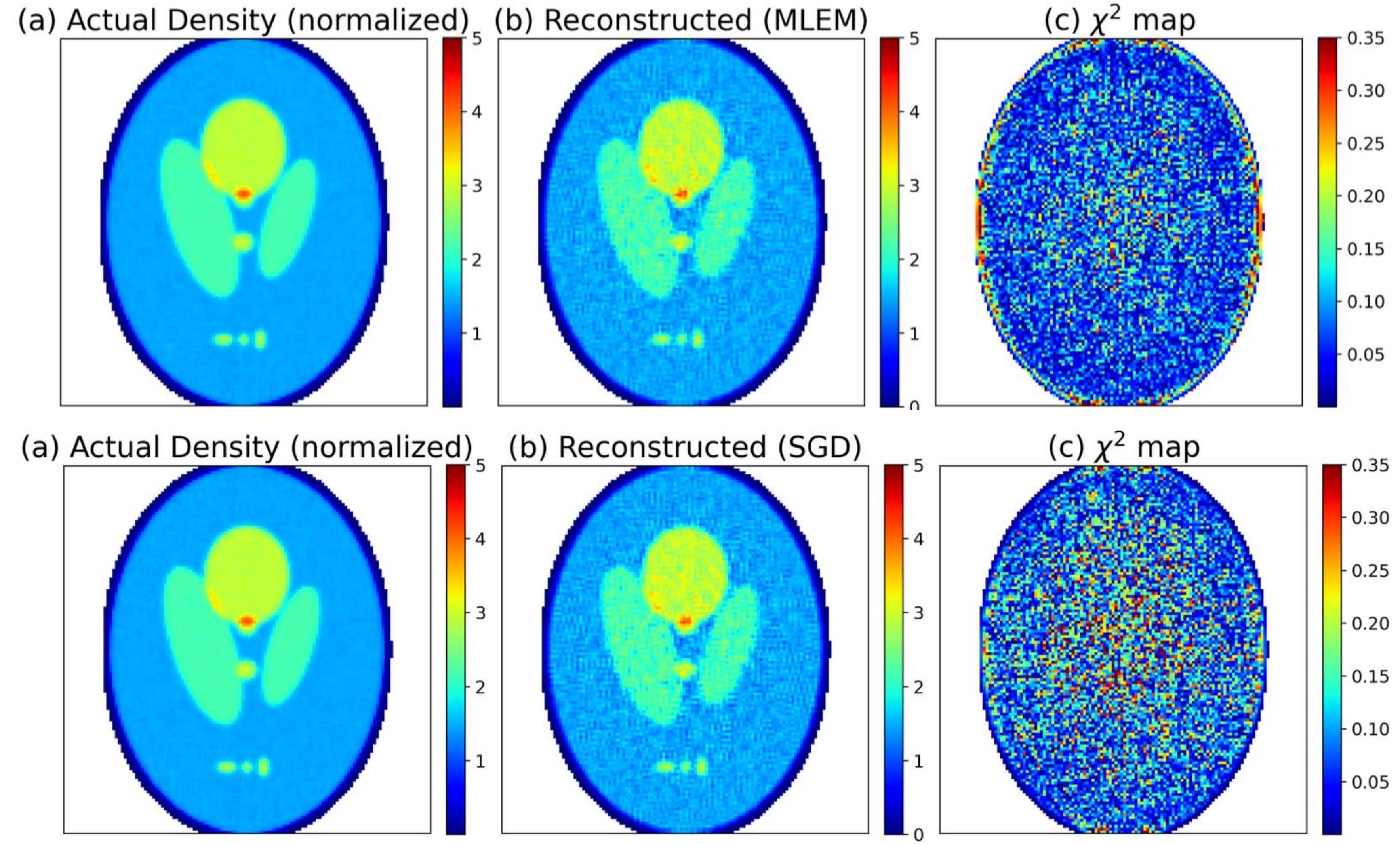


Differences between actual and reconstructed

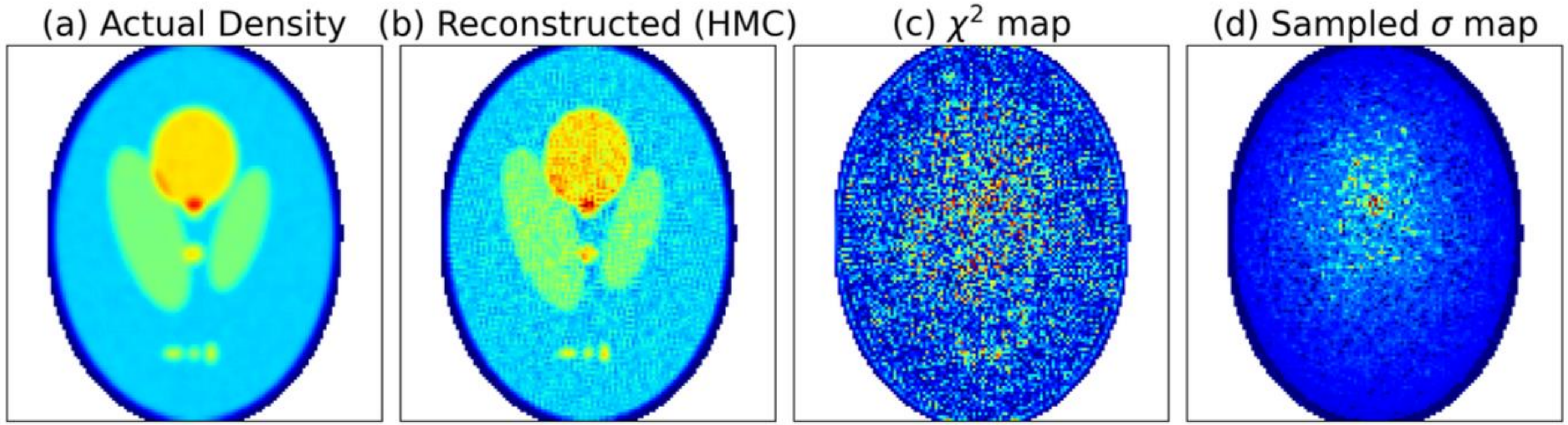




Equivalent performance to standard iterative methods

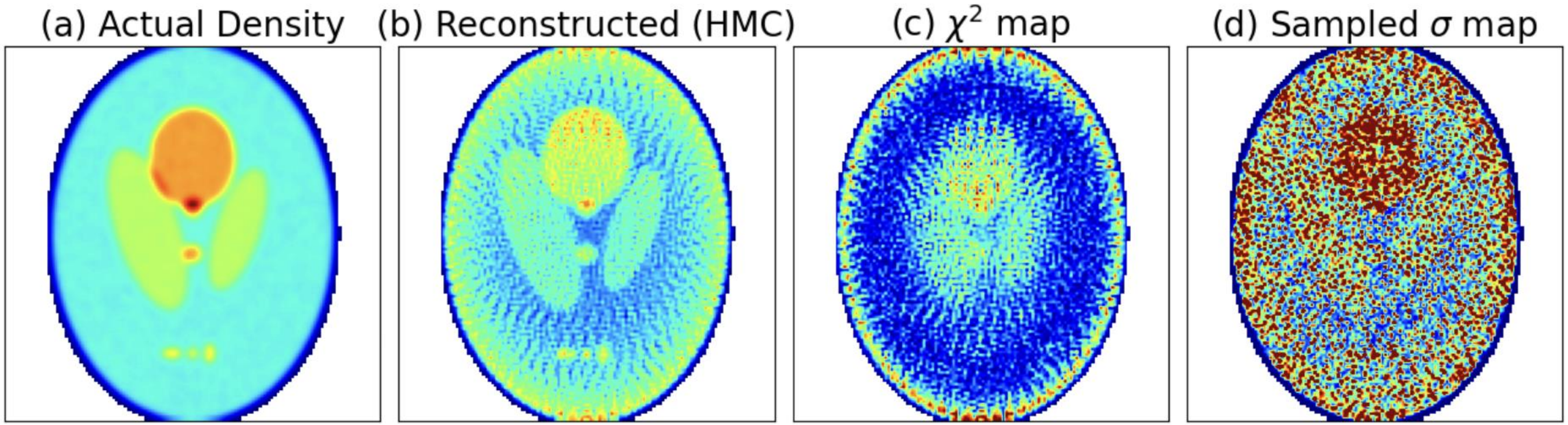


Stochastic Methods reflect the data's statistical nature



HMC reconstruction using an attenuation-corrected projection matrix.
(a) Ground truth activity distribution.
(b) Reconstructed activity (expectation value over the sampled ensemble).
(c) Voxel-wise χ2 map, indicating the local agreement between data and model.
(d) Voxel-wise σ map (standard deviation), reflecting the uncertainty in the reconstruction.

Stochastic Methods reflect the data's statistical nature



HMC reconstruction using a non-corrected projection matrix.

(a) Ground truth activity distribution.

(b) Reconstructed activity (expectation value over the sampled ensemble).

(c) Voxel-wise χ^2 map, indicating the local agreement between data and model.

(d) Voxel-wise σ map (standard deviation), reflecting the uncertainty in the reconstruction.

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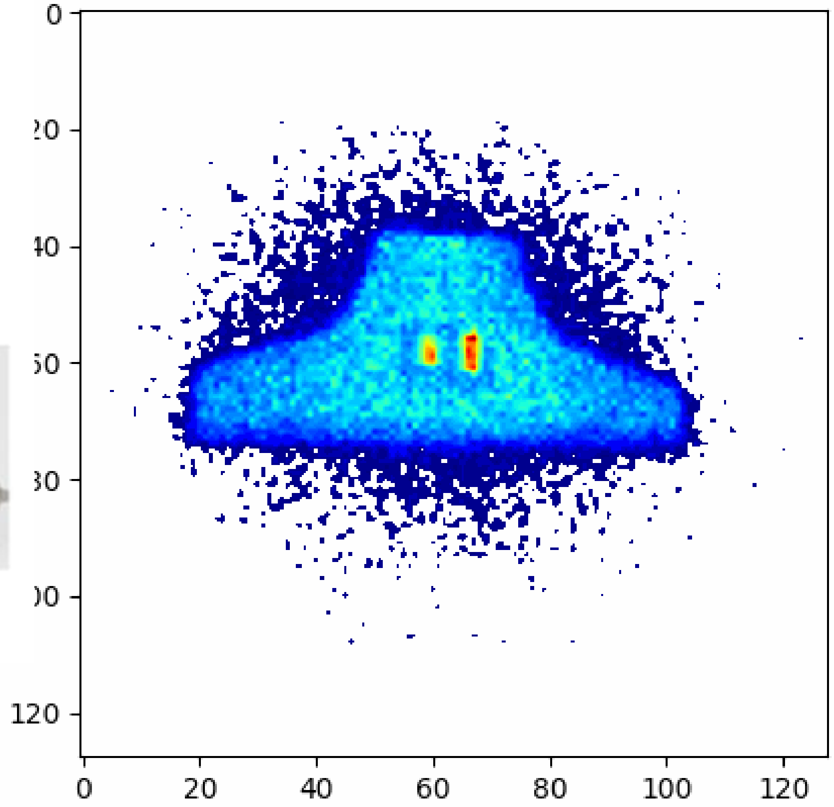
Stochastic Implementation

Application to Thyroid Phantom

Hardware Phantom

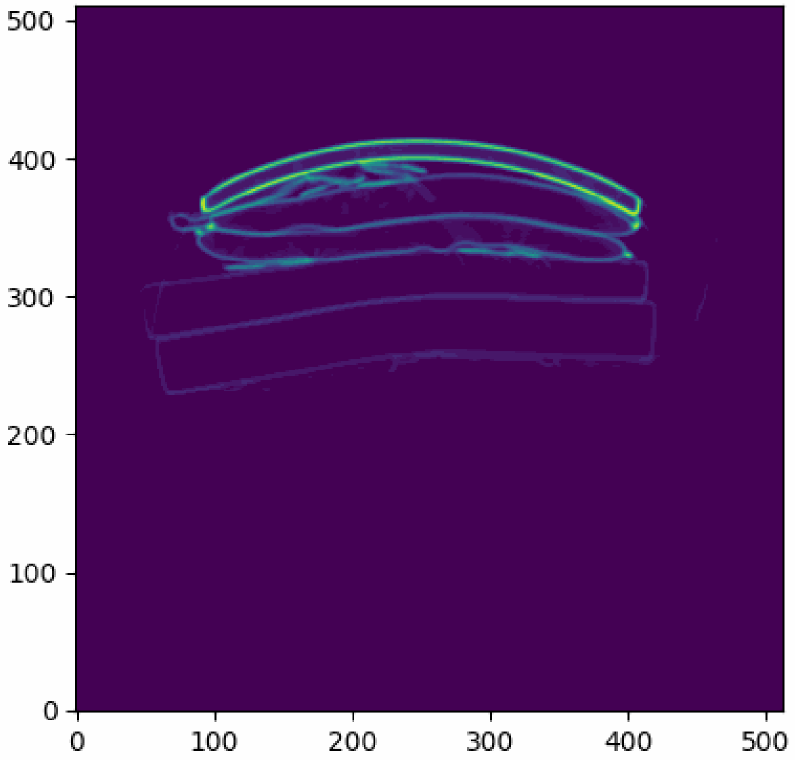


A Neck-Thyroid Phantom with Small Sizes of Thyroid Remnants for Postsurgical I-123 and I-131 SPECT/CT Imaging

by Konstantinos Michael [1,2,*], Anastasia Hadjiconstanti [3,†], Antonis Lontos [1,3], George Demosthenous [1], Savvas Frangos [4] and Yiannis Parpottas [1,3,*]

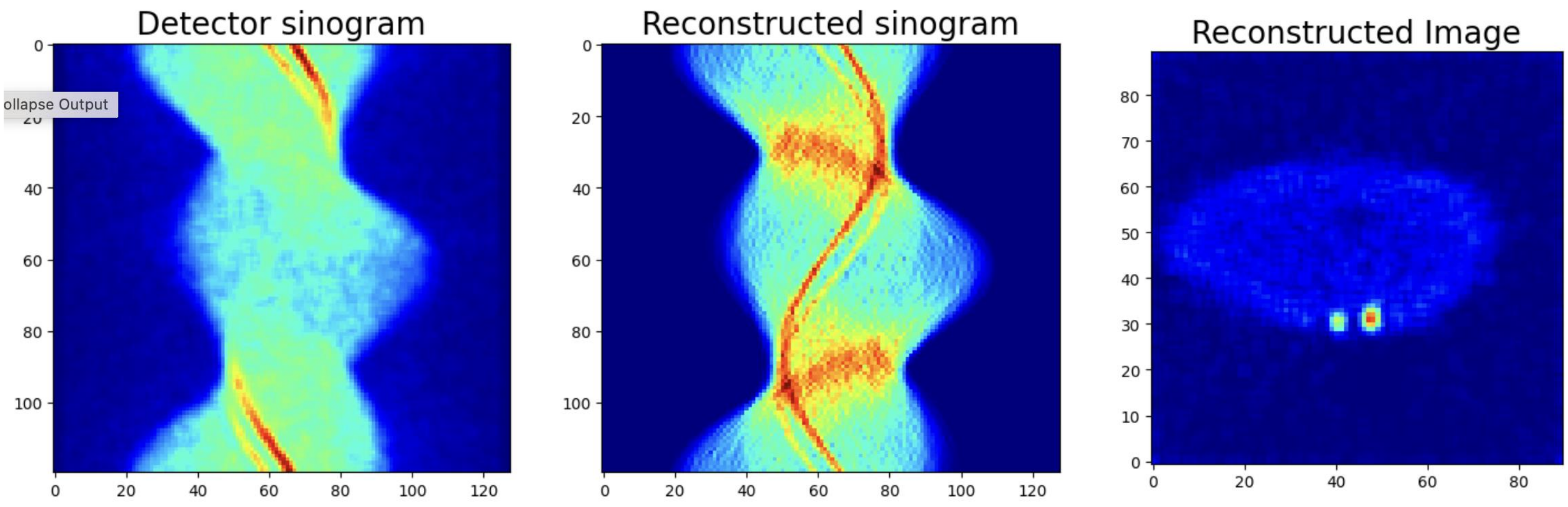
Life **2023**, *13*(4), 961; https://doi.org/10.3390/life13040961

Hardware Phantom



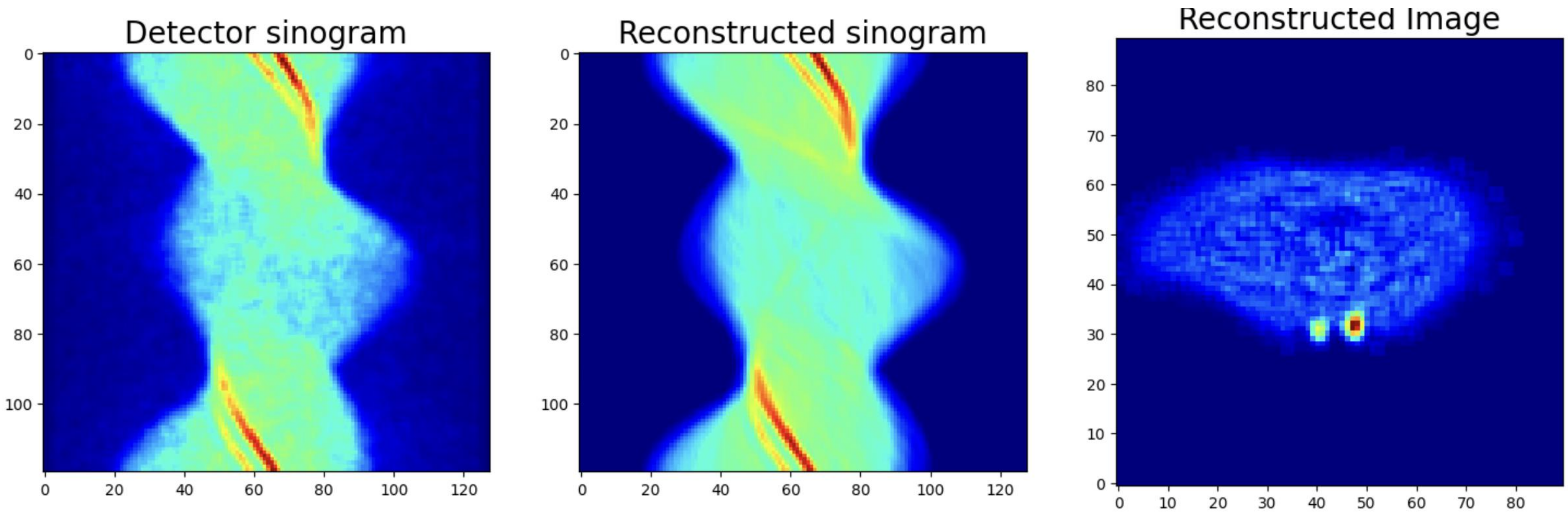
- SPECT/CT
- CT scan is used to provide the Attenuation Map

Hardware Phantom



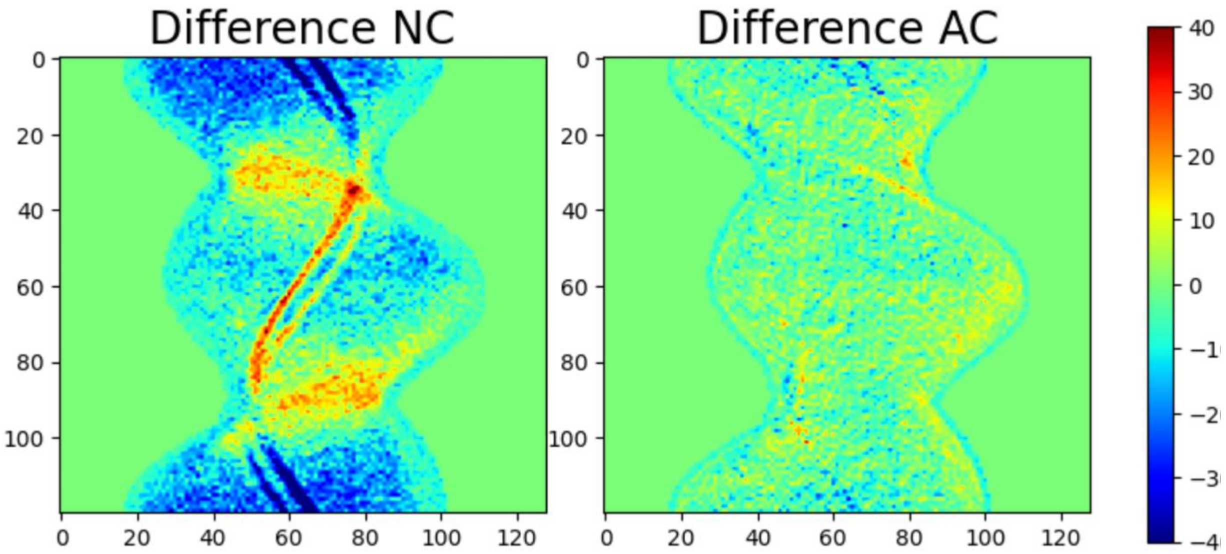
- Non-corrected Reconstruction

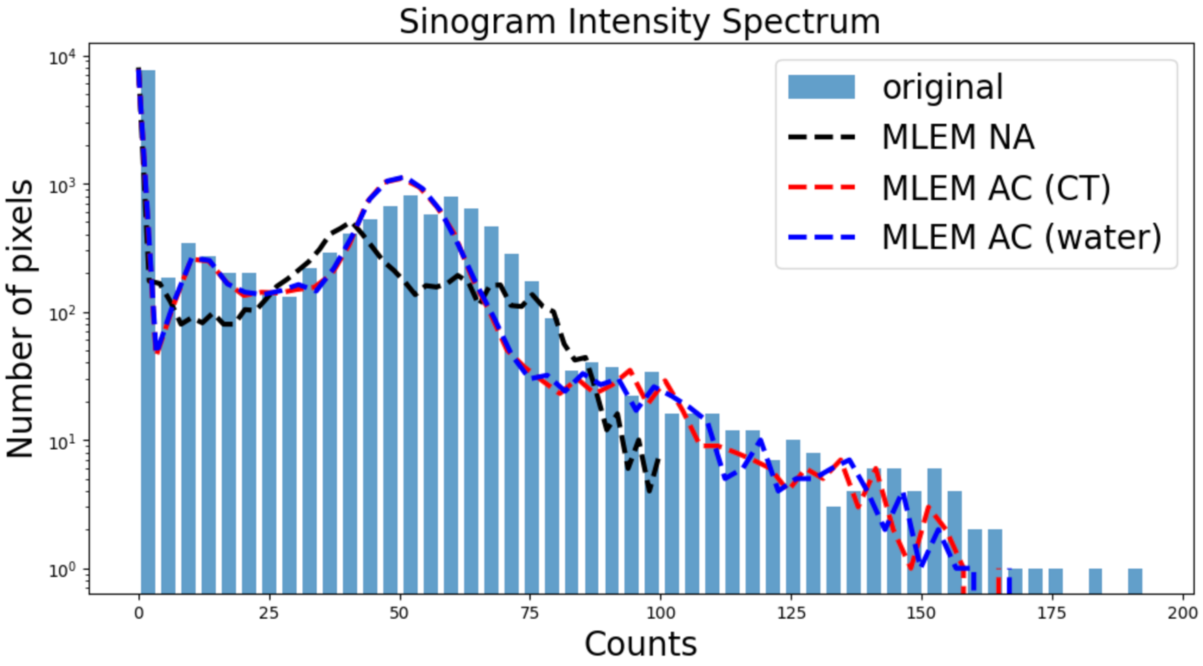
Hardware Phantom



- Attenuation-corrected Reconstruction
- Only primary photons

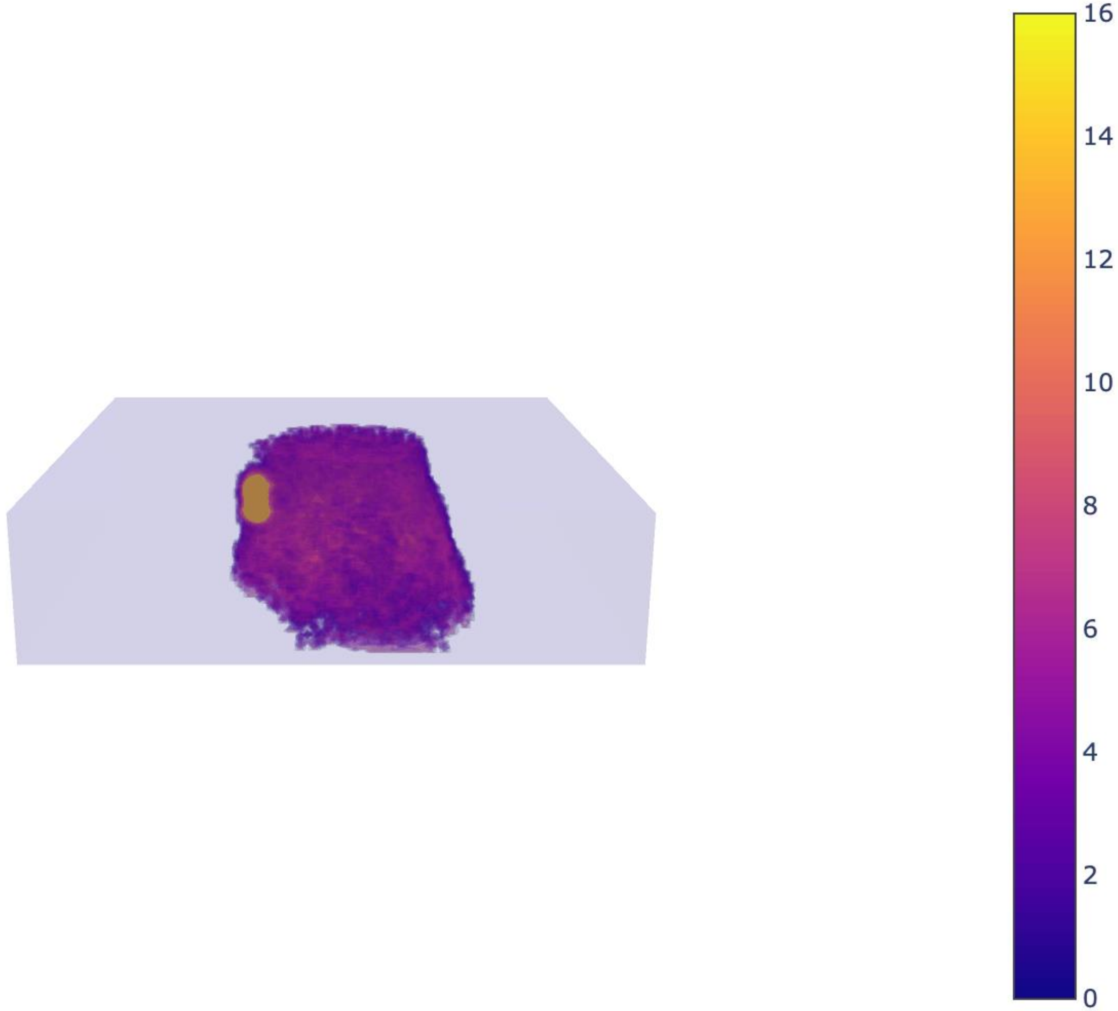
Hardware Phantom





- Reconstructed sinogram signifigantly improved
- Attenuation correction reproduces correct statistics

Hardware Phantom



- Reconstructed 3D image
- Corrected activity for each remnant compared to background
- Obtain estimates for the actual volume of each remnant

Conclusions

- Emission Tomography requires accurate modelling of all physical processes
- Nowadays ther exist practical alternativies to traditional reconstruction methods
- Our aim is to incorporate these new possibilities in order to enhance SPECT tomography
- Our methodology can be extended to other modalities as well